## Національний технічний університет України

**“Київський політехнічний інститут”**

**КАФЕДРА ЕЛЕКТРОННИХ ПРИЛАДІВ ТА ПРИСТРОЇВ**

**Реферат**

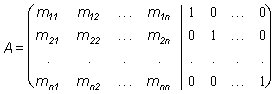
**з обчислювальної математики на тему :**

**Обращение матрицы с помощью расширенной матрицы**

**Перевірив:**

***Обращение матрицы с помощью расширенной матрицы***

Процедура обращает квадратную матрицу *M* размером *n\*n* с помощью элементарных операций, которые приводят матрицу *M* к единичной. Обозначим расширенную матрицу *A*:



К числу элементарных операций относятся:

1. Перестановка двух столбцов (строк) матрицы.
2. Умножение строки (столбца) на *k* не равное 0.
3. Сложение двух строк (столбцов).

Поскольку если матрица вырожденна, то у нее не существует обратной в алгоритме вводится дополнительная переменная *S*, по значению которой можно определить вырождена матрица (*S=1*) или нет (*S=0*).

***Блок-Схема***

***рис.1***

1

i=0;i<n;i++

2

j=0;j<n;j++

8

7

6

5

4

3

j=0;j<n;j++

Rassh(A,n,B,X)

j=i

Алгоритм нахождения обратной матрицы представлен в виде блок-схемы на

рис. 1. Блоки 2–5 отражают формирование столбца единичной матрицы. Если

условие 3 выполняется и элемент находится на главной диагонали, то он равен

единице, все остальные элементы нулевые. В блоке 6 происходит вызов

подпрограммы для решения системы уравнений методом Гаусса. В качестве

параметров в эту подпрограмму передается исходная матрица А,

сформированный в пунктах 2–5 вектор свободных коэффициентов В, размерность системы n. Вектор X будет решением *i-ой* системы уравнений и, следовательно, *i-ым* столбцом искомой матрицы Y.

## *Обращение матрицы методом Гаусса.*

Процедура находит, обратную квадратной матрице *A* размером *n\*n*, по методу Гаусса. Для несобственной матрицы *A=(ai j)* находится матрица *A-1=(xi j)* , такая, что

*A A -1=E,*

где *E*- единичная матрица.

Уравнение представляет собой *n* систем *n* линейных уравнений для *n2* неизвестных *xi j*. Каждая из систем имеет одну и ту же основную матрицу *A* и различные свободные члены. Все системы решаются одновременно методом Гаусса (см. [метод Гаусса](http://ftp.forsys.ru/literature/alg_lib/linalg/#gauss)).

В процедуре введена переменная *S*, если матрица близка к вырожденной, то *S*=1 и обратная матрица не вычисляется, иначе *S*=0.

***Пример***

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| Исходная матрица А. |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| A = | http://www.reshmat.ru/images/znak_matrix1.gif | |  |  |  | | --- | --- | --- | |  | 3 |  | | |  |  |  | | --- | --- | --- | | - | 1 |  | | |  |  |  | | --- | --- | --- | |  | 0 |  | | http://www.reshmat.ru/images/znak_matrix2.gif |  |
| |  |  |  | | --- | --- | --- | | - | 2 |  | | |  |  |  | | --- | --- | --- | |  | 1 |  | | |  |  |  | | --- | --- | --- | |  | 1 |  | |
| |  |  |  | | --- | --- | --- | |  | 2 |  | | |  |  |  | | --- | --- | --- | | - | 1 |  | | |  |  |  | | --- | --- | --- | |  | 4 |  | |

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| Найдем матрицу А-1 обратную к матрице А. |

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| Для этого напишем расширенную матрицу , в левой части которой находится наша исходная матрица А, а в правой единичная. |

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| Применяя метод Гаусса, последовательно будем приводить нашу исходную матрицу (левую часть расширенной матрицы) к единичной матрице. Причем совершенные преобразование мы будем применять ко всей расширенной матрице. |

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| Приведя левую часть расширенной матрицы к единичной, правая часть будет являться обратной матрицей к нашей исходной. |

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| Последовательность приведения левой части расширенной матрицы к единичной, Вы можете проследить по выделенным серыми прямоугольниками элементам. |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| http://www.reshmat.ru/images/znak_matrix1.gif | |  |  |  | | --- | --- | --- | |  | 3 |  | | |  |  |  | | --- | --- | --- | | - | 1 |  | | |  |  |  | | --- | --- | --- | |  | 0 |  | | http://www.reshmat.ru/images/znak_matrix3.gif | |  |  |  | | --- | --- | --- | |  | 1 |  | | |  |  |  | | --- | --- | --- | |  | 0 |  | | |  |  |  | | --- | --- | --- | |  | 0 |  | | http://www.reshmat.ru/images/znak_matrix2.gif |
| |  |  |  | | --- | --- | --- | | - | 2 |  | | |  |  |  | | --- | --- | --- | |  | 1 |  | | |  |  |  | | --- | --- | --- | |  | 1 |  | | |  |  |  | | --- | --- | --- | |  | 0 |  | | |  |  |  | | --- | --- | --- | |  | 1 |  | | |  |  |  | | --- | --- | --- | |  | 0 |  | |
| |  |  |  | | --- | --- | --- | |  | 2 |  | | |  |  |  | | --- | --- | --- | | - | 1 |  | | |  |  |  | | --- | --- | --- | |  | 4 |  | | |  |  |  | | --- | --- | --- | |  | 0 |  | | |  |  |  | | --- | --- | --- | |  | 0 |  | | |  |  |  | | --- | --- | --- | |  | 1 |  | |

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|  Рассмотрим столбец 1. |

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| Постараемся выполнять преобразования матрицы в целых числах. Поступим следующим образом: |

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| К элементам строки 1 прибавим соответствующие элементы строки 2. |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| http://www.reshmat.ru/images/znak_matrix1.gif | |  |  |  | | --- | --- | --- | |  | 1 |  | | |  |  |  | | --- | --- | --- | |  | 0 |  | | |  |  |  | | --- | --- | --- | |  | 1 |  | | http://www.reshmat.ru/images/znak_matrix3.gif | |  |  |  | | --- | --- | --- | |  | 1 |  | | |  |  |  | | --- | --- | --- | |  | 1 |  | | |  |  |  | | --- | --- | --- | |  | 0 |  | | http://www.reshmat.ru/images/znak_matrix2.gif |
| |  |  |  | | --- | --- | --- | | - | 2 |  | | |  |  |  | | --- | --- | --- | |  | 1 |  | | |  |  |  | | --- | --- | --- | |  | 1 |  | | |  |  |  | | --- | --- | --- | |  | 0 |  | | |  |  |  | | --- | --- | --- | |  | 1 |  | | |  |  |  | | --- | --- | --- | |  | 0 |  | |
| |  |  |  | | --- | --- | --- | |  | 2 |  | | |  |  |  | | --- | --- | --- | | - | 1 |  | | |  |  |  | | --- | --- | --- | |  | 4 |  | | |  |  |  | | --- | --- | --- | |  | 0 |  | | |  |  |  | | --- | --- | --- | |  | 0 |  | | |  |  |  | | --- | --- | --- | |  | 1 |  | |

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| К элементам стороки 2 прибавим соответствующие элементы строки 1 умноженные на 2. |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| http://www.reshmat.ru/images/znak_matrix1.gif | |  |  |  | | --- | --- | --- | |  | 1 |  | | |  |  |  | | --- | --- | --- | |  | 0 |  | | |  |  |  | | --- | --- | --- | |  | 1 |  | | http://www.reshmat.ru/images/znak_matrix3.gif | |  |  |  | | --- | --- | --- | |  | 1 |  | | |  |  |  | | --- | --- | --- | |  | 1 |  | | |  |  |  | | --- | --- | --- | |  | 0 |  | | http://www.reshmat.ru/images/znak_matrix2.gif |
| |  |  |  | | --- | --- | --- | |  | 0 |  | | |  |  |  | | --- | --- | --- | |  | 1 |  | | |  |  |  | | --- | --- | --- | |  | 3 |  | | |  |  |  | | --- | --- | --- | |  | 2 |  | | |  |  |  | | --- | --- | --- | |  | 3 |  | | |  |  |  | | --- | --- | --- | |  | 0 |  | |
| |  |  |  | | --- | --- | --- | |  | 2 |  | | |  |  |  | | --- | --- | --- | | - | 1 |  | | |  |  |  | | --- | --- | --- | |  | 4 |  | | |  |  |  | | --- | --- | --- | |  | 0 |  | | |  |  |  | | --- | --- | --- | |  | 0 |  | | |  |  |  | | --- | --- | --- | |  | 1 |  | |

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| К элементам стороки 3 прибавим соответствующие элементы строки 1 умноженные на -2. |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| http://www.reshmat.ru/images/znak_matrix1.gif | |  |  |  | | --- | --- | --- | |  | 1 |  | | |  |  |  | | --- | --- | --- | |  | 0 |  | | |  |  |  | | --- | --- | --- | |  | 1 |  | | http://www.reshmat.ru/images/znak_matrix3.gif | |  |  |  | | --- | --- | --- | |  | 1 |  | | |  |  |  | | --- | --- | --- | |  | 1 |  | | |  |  |  | | --- | --- | --- | |  | 0 |  | | http://www.reshmat.ru/images/znak_matrix2.gif |
| |  |  |  | | --- | --- | --- | |  | 0 |  | | |  |  |  | | --- | --- | --- | |  | 1 |  | | |  |  |  | | --- | --- | --- | |  | 3 |  | | |  |  |  | | --- | --- | --- | |  | 2 |  | | |  |  |  | | --- | --- | --- | |  | 3 |  | | |  |  |  | | --- | --- | --- | |  | 0 |  | |
| |  |  |  | | --- | --- | --- | |  | 0 |  | | |  |  |  | | --- | --- | --- | | - | 1 |  | | |  |  |  | | --- | --- | --- | |  | 2 |  | | |  |  |  | | --- | --- | --- | | - | 2 |  | | |  |  |  | | --- | --- | --- | | - | 2 |  | | |  |  |  | | --- | --- | --- | |  | 1 |  | |

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|  Рассмотрим столбец 2. |

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| Поменяем местами строки   2   и   3 . |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| http://www.reshmat.ru/images/znak_matrix1.gif | |  |  |  | | --- | --- | --- | |  | 1 |  | | |  |  |  | | --- | --- | --- | |  | 0 |  | | |  |  |  | | --- | --- | --- | |  | 1 |  | | http://www.reshmat.ru/images/znak_matrix3.gif | |  |  |  | | --- | --- | --- | |  | 1 |  | | |  |  |  | | --- | --- | --- | |  | 1 |  | | |  |  |  | | --- | --- | --- | |  | 0 |  | | http://www.reshmat.ru/images/znak_matrix2.gif |
| |  |  |  | | --- | --- | --- | |  | 0 |  | | |  |  |  | | --- | --- | --- | | - | 1 |  | | |  |  |  | | --- | --- | --- | |  | 2 |  | | |  |  |  | | --- | --- | --- | | - | 2 |  | | |  |  |  | | --- | --- | --- | | - | 2 |  | | |  |  |  | | --- | --- | --- | |  | 1 |  | |
| |  |  |  | | --- | --- | --- | |  | 0 |  | | |  |  |  | | --- | --- | --- | |  | 1 |  | | |  |  |  | | --- | --- | --- | |  | 3 |  | | |  |  |  | | --- | --- | --- | |  | 2 |  | | |  |  |  | | --- | --- | --- | |  | 3 |  | | |  |  |  | | --- | --- | --- | |  | 0 |  | |

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| К элементам строки 3 прибавим соответствующие элементы строки 2. |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| http://www.reshmat.ru/images/znak_matrix1.gif | |  |  |  | | --- | --- | --- | |  | 1 |  | | |  |  |  | | --- | --- | --- | |  | 0 |  | | |  |  |  | | --- | --- | --- | |  | 1 |  | | http://www.reshmat.ru/images/znak_matrix3.gif | |  |  |  | | --- | --- | --- | |  | 1 |  | | |  |  |  | | --- | --- | --- | |  | 1 |  | | |  |  |  | | --- | --- | --- | |  | 0 |  | | http://www.reshmat.ru/images/znak_matrix2.gif |
| |  |  |  | | --- | --- | --- | |  | 0 |  | | |  |  |  | | --- | --- | --- | | - | 1 |  | | |  |  |  | | --- | --- | --- | |  | 2 |  | | |  |  |  | | --- | --- | --- | | - | 2 |  | | |  |  |  | | --- | --- | --- | | - | 2 |  | | |  |  |  | | --- | --- | --- | |  | 1 |  | |
| |  |  |  | | --- | --- | --- | |  | 0 |  | | |  |  |  | | --- | --- | --- | |  | 0 |  | | |  |  |  | | --- | --- | --- | |  | 5 |  | | |  |  |  | | --- | --- | --- | |  | 0 |  | | |  |  |  | | --- | --- | --- | |  | 1 |  | | |  |  |  | | --- | --- | --- | |  | 1 |  | |

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|  Рассмотрим столбец 3. |

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| К элементам стороки 1 прибавим соответствующие элементы строки 3 умноженные на -1/5. |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| http://www.reshmat.ru/images/znak_matrix1.gif | |  |  |  | | --- | --- | --- | |  | 1 |  | | |  |  |  | | --- | --- | --- | |  | 0 |  | | |  |  |  | | --- | --- | --- | |  | 0 |  | | http://www.reshmat.ru/images/znak_matrix3.gif | |  |  |  | | --- | --- | --- | |  | 1 |  | | |  |  |  | | --- | --- | --- | |  | 4 |  | |  | | 5 | | |  |  |  | | --- | --- | --- | | - | 1 |  | |  | | 5 | | http://www.reshmat.ru/images/znak_matrix2.gif |
| |  |  |  | | --- | --- | --- | |  | 0 |  | | |  |  |  | | --- | --- | --- | | - | 1 |  | | |  |  |  | | --- | --- | --- | |  | 2 |  | | |  |  |  | | --- | --- | --- | | - | 2 |  | | |  |  |  | | --- | --- | --- | | - | 2 |  | | |  |  |  | | --- | --- | --- | |  | 1 |  | |
| |  |  |  | | --- | --- | --- | |  | 0 |  | | |  |  |  | | --- | --- | --- | |  | 0 |  | | |  |  |  | | --- | --- | --- | |  | 5 |  | | |  |  |  | | --- | --- | --- | |  | 0 |  | | |  |  |  | | --- | --- | --- | |  | 1 |  | | |  |  |  | | --- | --- | --- | |  | 1 |  | |

|  |
| --- |
| К элементам стороки 2 прибавим соответствующие элементы строки 3 умноженные на -2/5. |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| http://www.reshmat.ru/images/znak_matrix1.gif | |  |  |  | | --- | --- | --- | |  | 1 |  | | |  |  |  | | --- | --- | --- | |  | 0 |  | | |  |  |  | | --- | --- | --- | |  | 0 |  | | http://www.reshmat.ru/images/znak_matrix3.gif | |  |  |  | | --- | --- | --- | |  | 1 |  | | |  |  |  | | --- | --- | --- | |  | 4 |  | |  | | 5 | | |  |  |  | | --- | --- | --- | | - | 1 |  | |  | | 5 | | http://www.reshmat.ru/images/znak_matrix2.gif |
| |  |  |  | | --- | --- | --- | |  | 0 |  | | |  |  |  | | --- | --- | --- | | - | 1 |  | | |  |  |  | | --- | --- | --- | |  | 0 |  | | |  |  |  | | --- | --- | --- | | - | 2 |  | | |  |  |  | | --- | --- | --- | | - | 12 |  | |  | | 5 | | |  |  |  | | --- | --- | --- | |  | 3 |  | |  | | 5 | |
| |  |  |  | | --- | --- | --- | |  | 0 |  | | |  |  |  | | --- | --- | --- | |  | 0 |  | | |  |  |  | | --- | --- | --- | |  | 5 |  | | |  |  |  | | --- | --- | --- | |  | 0 |  | | |  |  |  | | --- | --- | --- | |  | 1 |  | | |  |  |  | | --- | --- | --- | |  | 1 |  | |

|  |
| --- |
| Элементы строки 2 разделим на    -1 . |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| http://www.reshmat.ru/images/znak_matrix1.gif | |  |  |  | | --- | --- | --- | |  | 1 |  | | |  |  |  | | --- | --- | --- | |  | 0 |  | | |  |  |  | | --- | --- | --- | |  | 0 |  | | http://www.reshmat.ru/images/znak_matrix3.gif | |  |  |  | | --- | --- | --- | |  | 1 |  | | |  |  |  | | --- | --- | --- | |  | 4 |  | |  | | 5 | | |  |  |  | | --- | --- | --- | | - | 1 |  | |  | | 5 | | http://www.reshmat.ru/images/znak_matrix2.gif |
| |  |  |  | | --- | --- | --- | |  | 0 |  | | |  |  |  | | --- | --- | --- | |  | 1 |  | | |  |  |  | | --- | --- | --- | |  | 0 |  | | |  |  |  | | --- | --- | --- | |  | 2 |  | | |  |  |  | | --- | --- | --- | |  | 12 |  | |  | | 5 | | |  |  |  | | --- | --- | --- | | - | 3 |  | |  | | 5 | |
| |  |  |  | | --- | --- | --- | |  | 0 |  | | |  |  |  | | --- | --- | --- | |  | 0 |  | | |  |  |  | | --- | --- | --- | |  | 5 |  | | |  |  |  | | --- | --- | --- | |  | 0 |  | | |  |  |  | | --- | --- | --- | |  | 1 |  | | |  |  |  | | --- | --- | --- | |  | 1 |  | |

|  |
| --- |
| Элементы строки 3 разделим на    5 . |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| http://www.reshmat.ru/images/znak_matrix1.gif | |  |  |  | | --- | --- | --- | |  | 1 |  | | |  |  |  | | --- | --- | --- | |  | 0 |  | | |  |  |  | | --- | --- | --- | |  | 0 |  | | http://www.reshmat.ru/images/znak_matrix3.gif | |  |  |  | | --- | --- | --- | |  | 1 |  | | |  |  |  | | --- | --- | --- | |  | 4 |  | |  | | 5 | | |  |  |  | | --- | --- | --- | | - | 1 |  | |  | | 5 | | http://www.reshmat.ru/images/znak_matrix2.gif |
| |  |  |  | | --- | --- | --- | |  | 0 |  | | |  |  |  | | --- | --- | --- | |  | 1 |  | | |  |  |  | | --- | --- | --- | |  | 0 |  | | |  |  |  | | --- | --- | --- | |  | 2 |  | | |  |  |  | | --- | --- | --- | |  | 12 |  | |  | | 5 | | |  |  |  | | --- | --- | --- | | - | 3 |  | |  | | 5 | |
| |  |  |  | | --- | --- | --- | |  | 0 |  | | |  |  |  | | --- | --- | --- | |  | 0 |  | | |  |  |  | | --- | --- | --- | |  | 1 |  | | |  |  |  | | --- | --- | --- | |  | 0 |  | | |  |  |  | | --- | --- | --- | |  | 1 |  | |  | | 5 | | |  |  |  | | --- | --- | --- | |  | 1 |  | |  | | 5 | |

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| --- |
| Ответ : |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| A-1 = | http://www.reshmat.ru/images/znak_matrix1.gif | |  |  |  | | --- | --- | --- | |  | 1 |  | | |  |  |  | | --- | --- | --- | |  | 4 |  | |  | | 5 | | |  |  |  | | --- | --- | --- | | - | 1 |  | |  | | 5 | | http://www.reshmat.ru/images/znak_matrix2.gif |  |
| |  |  |  | | --- | --- | --- | |  | 2 |  | | |  |  |  | | --- | --- | --- | |  | 12 |  | |  | | 5 | | |  |  |  | | --- | --- | --- | | - | 3 |  | |  | | 5 | |
| |  |  |  | | --- | --- | --- | |  | 0 |  | | |  |  |  | | --- | --- | --- | |  | 1 |  | |  | | 5 | | |  |  |  | | --- | --- | --- | |  | 1 |  | |  | | 5 | |

***Пример в маткад***





***Вывод***

Сравнив все методы я понял, что метод Гаусса наиболее прост в его использовании и он широко применяется по сравнении с другими методами.

***Источники:***

<http://www.reshmat.ru>

http://ftp.forsys.ru